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Dynamics of a Fluid Conveying Fiber-Reinforced Shell

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Nomenclature

a	= shell radius
A_{ij}	= elements of stretching stiffness matrix
B_{ij}	= elements of bending-stretching stiffness matrix
D_{ij}	= elements of bending stiffness matrix
$\bar{A}_{ij},\bar{B}_{ij},\bar{D}_{ij}$	$= \frac{1}{A_{22}}\left(A_{ij},\frac{B_{ij}}{a},\frac{D_{ij}}{a^2}\right)$
E_{11},E_{22},G_{12}	= orthotropic elastic constants
h	= thickness of shell
I_n	= modified Bessel function of first kind of order n
K_n	= modified Bessel function of second kind of order n
ℓ	= shell length
m,n	= number of axial half-waves and circumferential waves
p	= perturbation pressure
q_i	= surface loads on the shell
t	= time
u_1,u_2,u_3	= axial, circumferential and radial displacements
x,θ,r	= axial, circumferential and radial coordinates
$\bar{V}=V/(E_{11}/\rho_s)^{1/2}$	= dimensionless fluid velocity
α	$=x/a$
β	$=m\pi/\ell$
γ	= orientation of fiber axes relative to structural axes
ν	= major Poisson's ratio
δ_{ij}	= Kronecker delta
λ	$=m\pi a/\ell$
ρ_i,ρ_0	= mass density of fluid inside and outside of the shell
ω	= circular frequency
ω_0	$=(E_{11}/\rho_s a^2)^{1/2}$
Ω	$=\omega/\omega_0$
ϕ	= perturbation velocity potential
η_B,η_H	= parameters defined in Eq. (3)

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Introduction

VIBRATION characteristics of circular, cylindrical, isotropic shells, containing a flowing fluid, have been studied by several investigators using beam-type theory.[1] Sufficiently high flow velocity reduces the natural frequency of bending vibration and finally causes a static divergence instability. Static instability for short, thin, isotropic shells is quite often associated with higher-order circumferential modes; hence shell theory has to be utilized to predict the divergence instability for these modes. A number of authors (primarily Weaver and Paidoussis) have noted that flutter of a short, thin, cylindrical shell is possible for all end conditions, if the flow velocity is sufficiently high.

Within the last few years several papers have been published concerning the problem of an anisotropic circular cylindrical shell containing a flowing fluid.[2-5] This paper is based on the work presented in Ref. 3. An analytical method is developed to account for the effects of inviscid, incompressible, and irrotational flow on the vibration characteristics of thin, cylindrical fiber-reinforced shells. Particular attention is paid to the determination of the natural frequency of vibration and the divergence boundaries for various circumferential modes n. The results are derived by utilizing Galerkin's method to solve the Flügge-type equations of motion. Two cases are considered here: a shell containing a flowing fluid and a fluid conveying shell which is surrounded by a static fluid.

Shell Equations

A thin anisotropic cylindrical shell of length ℓ and thickness h conveys a fluid with velocity V. The shell consists of N homogeneous, orthotropic lamina. Each layer has arbitrary thickness, elastic properties, and orientation. A Flügge-type equation of motion is developed in cylindrical coordinates (x,θ,r) neglecting axial and circumferential inertia forces.[6]

$$\sum_{j=1}^{3} L_{ij}u_j=\delta_{i3}q_i \quad i=1,2,3 \tag{1}$$

where the differential operators are functions of $\bar{A}_{ij},\bar{B}_{ij},\bar{D}_{ij}$, α, and θ.

Equation (1) represents a linear system of equations in u_1,u_2, and u_3, which can be reduced by Gaussian elimination to one equation in terms of the radial displacement u_3:

$$[L_{11}^2L_{22}L_{33}-L_{11}L_{12}^2L_{33}-L_{13}^2L_{22}L_{11}-L_{23}^2L_{11}^2 + 2L_{23}L_{11}L_{12}L_{13}]u_3=[L_{22}L_{11}^2-L_{12}^2L_{11}]q_3 \tag{2}$$

where q_3 includes both the inertia of the shell and the dynamic pressure on the shell due to fluid flow.

Fluid Equations

The governing equation for the flow is Laplace's equation, written in terms of the perturbation velocity potential. The perturbation pressure p is then given by the linearized unsteady Bernoulli equation for the case of a fluid conveying shell with and without a surrounding static fluid. Solution of this equation, subject to appropriate boundary conditions on the surface of the shell yields the following expression:

$$p = -\rho_i\eta_B\left(\frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x \partial t} + V^2\frac{\partial^2 w}{\partial x^2}\right) + \rho_0\eta_H\frac{\partial^2 w}{\partial t^2} \tag{3}$$

where

$$\eta_B = aI_n(\beta a)/[nI_n(\beta a) + \beta a I_{n+1}(\beta a)]; \quad w = u_3$$

$$\eta_H = aK_n(\beta a)/[nK_n(\beta a) - \beta a K_{n+1}(\beta a)]$$

Characteristic Equation

Adding the effect of the inertia of the shell to Eq. (3), an expression for q_3 can be written as

$$q_3 = -\frac{E_{11}h}{A_{22}}\left[\frac{1}{\omega_0^2}\left(1 + \frac{1}{\rho_s h}(\rho_i\eta_B - \rho_0\eta_H)\right)\frac{\partial^2 w}{\partial t^2} + \frac{1}{\omega_0\rho_s h}(2\rho_i\eta_B\bar{V})\frac{\partial^2 w}{\partial\alpha\partial t} + \frac{1}{\rho_s h}(\rho_i\eta_B\bar{V}^2)\frac{\partial^2 w}{\partial\alpha^2}\right] \tag{4}$$

The shell is assumed to be simply supported at both ends, and homogeneous initial displacements and velocities are imposed. A suitable solution may be written as

$$u_3 = w = \sum_{m=1}^{\infty} W_{mn}\exp(i\omega t)\sin(\lambda\alpha)\cos(n\theta) \tag{5}$$

Equations (4) and (5) are substituted into Eq. (2) and Galerkin's method is applied (see acknowledgment). An infinite set of linear homogeneous equation in W_{mn} is obtained:

$$\xi_1^{kn} W_{kn} + \sum_{m=1}^{\infty} \xi_2^{mn} \frac{4k}{(k^2 - m^2)\pi} W_{mn} = 0 \tag{6}$$

for $k = 1,2,3,4,\ldots$ and $m \pm k$ odd. The coefficients ξ_1^{kn} and ξ_2^{mn} are functions of $\bar{A}_{ij}, \bar{B}_{ij}, \bar{D}_{ij}, h, n, \bar{V}, \eta_B, \eta_H, \lambda, \rho_i, \rho_0, \rho_s$, and Ω.

A nontrivial solution is found by setting the determinant of the coefficients to zero. The characteristic equation for the natural frequency is then found as a function of the fluid flow velocity. At the point of instability the off-diagonal terms in the characteristic determinant can be shown to be zero, as the natural frequency goes to zero. The stability analysis of the system as a function of fluid flow velocity may then be made independently.

Numerical Results

The shell was assumed to have eight layers of ±45-deg orientation and a specific gravity of 2.0. The orthotropic constants were taken as $E_{11} = 51.7$ GN/m^2, $E_{22} = 24.1$ GN/m^2, $G_{12} = 8.6$ GN/m^2, and $\nu_{12} = 0.25$.

Figure 1 ($\ell/a = 5$, $h/a = 0.01$) shows the variation of natural frequency as a function of flow velocity, for given circumferential modes n. The case of a shell in a vacuum is considered as well as one that is surrounded by a static fluid. The natural frequencies for all circumferential modes, decrease to zero as the fluid velocity is increased. The lowest natural frequency and the earliest occurrence of static divergence instability, for the particular shell investigated, is associated with $n = 3$ circumferential waves.

Natural frequency as a function of the circumferential wave number for a given flow velocity is shown in Fig. 2 for $\ell/a = 5$ and $h/a = 0.01$. The $n = 3$ mode yields the greatest reduction in natural frequency for a particular fluid velocity and is the mode associated with the earliest divergence. The axisymmetric ($n = 0$) mode was not considered.

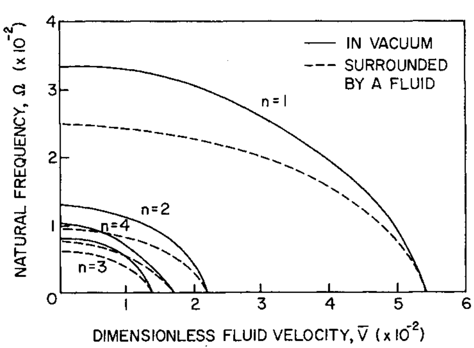

Fig. 1 Effect of fluid velocity on the natural frequency of a fluid conveying shell ($\ell/a = 5$, $h/a = 0.01$, $a = 20.32$ cm, $\omega_0 = 3982$ Hz, $V = 51$ m/s for $\bar{V} = 0.01$).

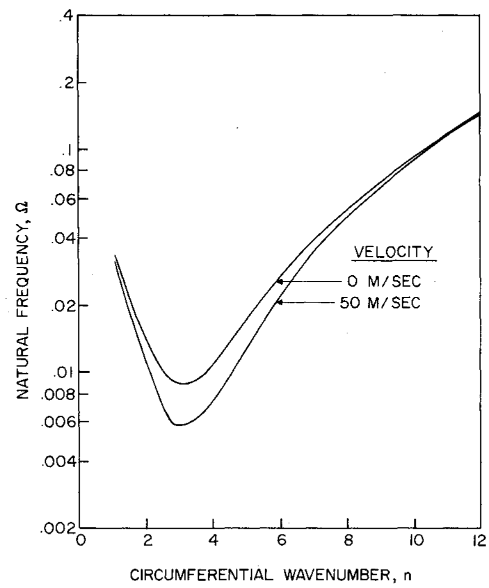

Fig. 2 Effect of circumferential wave number on the natural frequency of a fluid conveying shell ($\ell/a = 5$, $h/a = 0.01$, $a = 20.32$ cm, $\omega_0 = 3982$ Hz, $V = 51$ m/s for $\bar{V} = 0.01$).

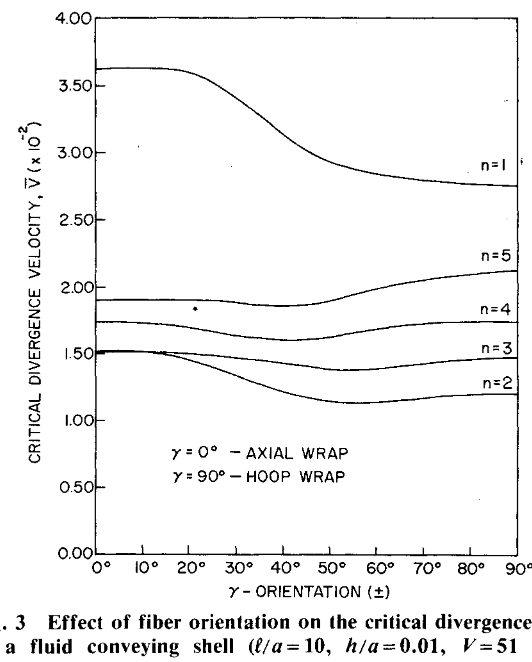

Fig. 3 Effect of fiber orientation on the critical divergence velocity of a fluid conveying shell ($\ell/a = 10$, $h/a = 0.01$, $V = 51$ m/s for $\bar{V} = 0.01$).

A minimum of four terms in the solution form was necessary to predict accurately the divergence velocities as a function of the length-to-radius ratio ℓ/a. Divergence velocities varied widely before approaching a limit as the number of terms in the assumed solution was increased. This was particularly true for $\ell/a>10$ and for higher-order modes ($n \geq 3$). A study of the effect of the thickness-to-radius ratio on critical divergence velocities showed that higher-order circumferential modes are associated with shorter and/or thinner shells. The lowest critical divergence velocity, for long shells ($\ell/a \geq 40$), is associated with the beam-type mode ($n=1$).

Critical divergence velocity as a function of lamina orientation, for a given circumferential mode, is shown in Fig. 3 for $\ell/a=10$ and $h/a=0.01$. The present analysis was repeated for the isotropic case and compared with the results given in Ref. 7. Good agreement was noted.

Conclusions

Natural frequencies of fiber-reinforced shells decrease with increasing fluid velocity, as do isotropic shells, until static divergence occurs. Natural frequency is a function of the length-to-radius ratio, the radius-to-thickness ratio, the circumferential mode number, and the orientation of the lamina. Beam-type theory was found to be suitable for long shells, but in any other case shell theory must be used. Qualitatively, the behavior of anisotropic shells appears to be much the same as that of isotropic shells. However, it is reasonable to expect that certain lamina stacking sequences will result in considerably different numerical values of divergence velocity as a function of lamina orientation.

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Stability of Inviscid Shear Flow over Flexible Membranes

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Introduction

THE shear flow over compliant surfaces is physically interesting and its linear stability has been treated exhaustively in the viscous case.[1,2] In this Note we consider the inviscid limit and give several results for self-excited disturbances. For various flow parameters these include bounds on unstable eigenvalues, sufficient conditions for stability, and explicit dispersion relations.

Analysis

We orient along the x axis the velocity $U(y)+u(x,y,t)$ and along the y axis the velocity $v(x,y,t)$ with t being time. Consider a flexible membrane whose deviation from equilibrium $y=0$ satisfies $y=n(x,t)$ and assume a rigid wall placed along $y=-H<0$ across which $v=0$. If $u,v \ll U$, the use of $u=\psi_y$, $v=-\psi_x$, and $\psi(x,y,t)=\phi(y)\exp ik(x-ct)$ leads to the linear Rayleigh equation:

$$(U-c)(\phi''-k^2\phi)-U''\phi=0 \tag{1}$$

where k is a specified wavenumber and $c=c_r+ic_i$ is the complex eigenvalue. At the wall

$$\phi(-H)=0 \tag{2}$$

Next, the disturbance pressure acting on the membrane satisfies $p(y=0)=\rho g n - T n_{xx} + sn + mn_{tt} + mdn_t$. Here we assume a fluid density ρ, a gravitational acceleration g, a membrane tension T, a spring constant s, a lineal mass density m, and a damping constant d. Assuming $p=\hat{p}(y)\exp ik(x-ct)$ and $n=a\exp ik(x-ct)$ leads to $\hat{p}(0)=(\rho g+Tk^2+s-mk^2c^2-imdkc)a$, while the kinematic condition $v=n_t+Un_x$ leads to $\phi(0)=-(U(0)-c)a$. Hence,

$$\frac{\phi'(0)}{\phi(0)}=\frac{U'(0)}{U(0)-c}+\frac{\rho g+Tk^2+s-mk^2c^2-imdkc}{\rho[U(0)-c]^2} \tag{3}$$

Equations (1-3) complete the inviscid stability formulation. Some general consequences will now be discussed.

First Howard's[3] semicircle theorem is extended to handle inhomogeneous boundary conditions. Euqation (1) and the definitions $\phi(y)=(c-U)F(y)$ and $W=U-c$ imply that $(W^2F')'-k^2W^2F=0$. Multiplication by the complex conjugate F^* and integration over $(-H,0)$ leads to

$$\int_{-H}^{0} W^2\{|F'|^2+k^2|F|^2\}dy=F^*(W^2F')|_{-H}^{0}=G|F_0|^2$$

where we have set $\rho G=\rho(G_r+iG_i)=\rho g+Tk^2+s-mk^2c^2-imdkc$, taken Eqs. (3) and (2) in the form $W^2F'=GF$ and $F(-H)=0$, and written $F_0=F(0)$. Now define $Q=|F'|^2+$

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